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**Determining the Weak Phase γ using the decays B_d ,
 $B^+ \rightarrow K\eta$ (η') and $B_s \rightarrow \pi\eta$ (η')**¹

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Abstract

We suggest two methods (based on flavor $SU(3)$ symmetry) to determine the CKM angle γ using the decays B_d , $B^+ \rightarrow K\eta$ (η') and $B_s \rightarrow \pi\eta$ (η'), respectively. Rescattering effects are partly included – we neglect annihilation amplitudes, but do not assume any other relation between the $SU(3)$ invariant amplitudes. We use the fact that the amplitude (including the Electroweak Penguin contribution) for B_d , $B^+ \rightarrow \pi K$ with final state I (isospin) $= 3/2$ is known as a function of γ from the decay rate $B^+ \rightarrow \pi^0\pi^+$.

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1 Introduction

The principal aim of the B physics experimental programs is to measure the angles (denoted by α , β and γ) of the triangle representing the unitarity relation: $V_{tb}^*V_{td} + V_{cb}^*V_{cd} + V_{ub}^*V_{ud} = 0$, where V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The idea is to overdetermine the angles of this triangle and thus test the CKM paradigm of CP violation.

Methods to determine γ ($\equiv \text{Arg}(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$) including the Electroweak Penguin (EWP) diagram contribution have been suggested [1, 2, 3, 4, 5]. To (over)determine γ to test the CKM theory of CP violation, we measure γ using different techniques. It is thus useful to have new methods to determine γ . With this motivation, in this letter, we give two new methods to determine γ using time *integrated* rates for the decays $B_d, B^+ \rightarrow K\eta$ (η') (Method 1) and $B_s \rightarrow \pi\eta$ (η') (Method 2). As in all the other methods mentioned above, flavor $SU(3)$ symmetry is used in both the methods.

We will write the decay amplitudes in terms of flavor $SU(3)$ invariant amplitudes [6]. These are denoted by $C_3^{T,P}$, $C_6^{T,P}$, $C_{15}^{T,P}$, $A_3^{T,P}$ and $A_{15}^{T,P}$ and correspond to the 5 linearly independent ways of forming flavor $SU(3)$ singlets from the initial meson B_i , the two final state mesons belonging to the flavor $SU(3)$ octet and the effective weak Hamiltonian which transforms as a $\bar{3} \times 3 \times \bar{3}$. T and P denote the parts of these amplitudes generated by tree level and penguin operators respectively. These invariant amplitudes include soft final state rescattering effects. Some of the methods to determine γ [4], including the ones which use the decays $B^+ \rightarrow K\eta$ (η') [2, 3] and the decay $B_s \rightarrow \pi\eta$ (η') [1] neglect rescattering effects. In particular, these methods assume that the decay amplitude $B^+ \rightarrow \pi^+ K^0$ has no weak phase $e^{i\gamma}$ from the tree level operators. In the language of the $SU(3)$ invariant amplitudes, this is equivalent to assuming that the annihilation amplitudes ⁴ A_i are suppressed by f_B/m_B and a combination of the $SU(3)$ invariant amplitudes, $C_3^T - C_6^T -$

⁴Annihilation amplitudes are the ones in which the index i of B_i is contracted directly with the Hamiltonian.

C_{15}^T is zero (both of which are valid in the absence of significant rescattering effects). Rescattering effects can enhance the annihilation contributions *and* lead to significant $C_3^T - C_6^T - C_{15}^T$ [7]. In this letter, we neglect annihilation contributions but *do not* assume any relation between C_3^T , C_6^T and C_{15}^T or the other $SU(3)$ invariant amplitudes. Thus, rescattering effects are partly included.

The decay amplitudes for $B_d \rightarrow \pi K$ can be written as [6]

$$\begin{aligned}
-\mathcal{A}(B_d \rightarrow \pi^- K^+) &= \lambda_u^{(s)}(C_3^T + C_6^T + 3C_{15}^T) + \sum_q \lambda_q^{(s)}(C_{3,q}^P + C_{6,q}^P + 3C_{15,q}^P) \\
&\quad - \lambda_u^{(s)} A_{15}^T - \sum_q \lambda_q^{(s)} A_{15,q}^P, \\
\sqrt{2} \mathcal{A}(B_d \rightarrow \pi^0 K^0) &= \lambda_u^{(s)}(C_3^T + C_6^T - 5C_{15}^T) + \sum_q \lambda_q^{(s)}(C_{3,q}^P + C_{6,q}^P - 5C_{15,q}^P) \\
&\quad - \lambda_u^{(s)} A_{15}^T - \sum_q \lambda_q^{(s)} A_{15,q}^P.
\end{aligned} \tag{1}$$

Here, $\lambda_q^{(q')} = V_{qb}^* V_{qq'}$ ($q = u, c, t$ and $q' = d, s$) and C_q^P , A_q^P denote the penguin amplitudes due to quark q running in the loop. Using the unitarity of the CKM matrix, *i.e.*, $\lambda_t^{(s)} = -\lambda_u^{(s)} - \lambda_c^{(s)}$, we get

$$\lambda_u^{(s)} C_i^T + \sum_q \lambda_q^{(s)} C_{i,q}^P = \lambda_u^{(s)} \tilde{C}_i^T - \lambda_c^{(s)} C_i^P, \tag{2}$$

where $\tilde{C}_i^T = C_i^T - C_{i,t}^P + C_{i,u}^P$ and $C_i^P = C_{i,t}^P - C_{i,c}^P$. A similar notation is used for \tilde{A}_i^T and A_i^P . Henceforth, we will write the decay amplitudes using this notation.

The decay amplitudes for $B^+ \rightarrow \pi K$ are [6]

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow \pi^+ K^0) &= \lambda_u^{(s)} \tilde{C}_3^T - \lambda_c^{(s)} C_3^P - \lambda_u^{(s)} \tilde{C}_6^T + \lambda_c^{(s)} C_6^P \\
&\quad - \lambda_u^{(s)} \tilde{C}_{15}^T + \lambda_c^{(s)} C_{15}^P + 3\lambda_u^{(s)} \tilde{A}_{15}^T - 3\lambda_c^{(s)} A_{15}^P, \\
-\sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+) &= \lambda_u^{(s)} \tilde{C}_3^T - \lambda_c^{(s)} C_3^P - \lambda_u^{(s)} \tilde{C}_6^T + \lambda_c^{(s)} C_6^P \\
&\quad + 7\lambda_u^{(s)} \tilde{C}_{15}^T - 7\lambda_c^{(s)} C_{15}^P + 3\lambda_u^{(s)} \tilde{A}_{15}^T - 3\lambda_c^{(s)} A_{15}^P.
\end{aligned} \tag{3}$$

Using Eqns.(1) and (3), we get an expression for $A_{3/2}$, the amplitude for B^+ , $B_d \rightarrow \pi K$ with final state I (isospin) $= 3/2$,

$$\begin{aligned} A_{3/2} &= \mathcal{A}(B^+ \rightarrow \pi^+ K^0) + \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+) \\ &= \mathcal{A}(B_d \rightarrow \pi^- K^+) + \sqrt{2} \mathcal{A}(B_d \rightarrow \pi^0 K^0) \\ &= -8 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right). \end{aligned} \quad (4)$$

The decay amplitude for $B^+ \rightarrow \pi^+ \pi^0$ is

$$- \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^+ \pi^0) = 8 \left(\lambda_u^{(d)} \tilde{C}_{15}^T - \lambda_c^{(d)} C_{15}^P \right). \quad (5)$$

The QCD penguin diagram (which is $\Delta I = 1/2$) does not contribute to this decay since this decay has the transition $\Delta I = 3/2$. So, C_{15}^P is the EWP contribution. Neubert, Rosner [8] showed that

$$C_{15,q}^P = C_{15}^T \frac{3}{2} \kappa_q, \quad (6)$$

where $\kappa_q = (c_{9,q} + c_{10,q})/(c_1 + c_2)$ is the ratio of Wilson coefficients (WC's) of the EWP operators (with quark q running in the loop) and the tree level operators in the effective Hamiltonian so that

$$- \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^+ \pi^0) = 8 C_{15}^T \left[\lambda_u^{(d)} \left(1 + \frac{3}{2} \kappa_u - \frac{3}{2} \kappa_t \right) - \lambda_c^{(d)} \left(\frac{3}{2} \kappa_t - \frac{3}{2} \kappa_c \right) \right]. \quad (7)$$

The top quark EWP diagram with Z exchange is enhanced by m_t^2/m_Z^2 and so $\kappa_t \gg \kappa_{u,c}$ giving

$$- \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^+ \pi^0) \approx 8 C_{15}^T \lambda_u^{(d)} \left[\left(1 - \frac{3}{2} \kappa \right) - \frac{\lambda_c^{(d)}}{\lambda_u^{(d)}} \frac{3}{2} \kappa \right], \quad (8)$$

where $\kappa = \kappa_t$. Since $3/2 \kappa \sim 2\%$ and $|\lambda_u^{(d)}| \sim |\lambda_c^{(d)}|$, we get

$$- \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^+ \pi^0) \approx 8 C_{15}^T |\lambda_u^{(d)}| e^{i\gamma} \quad (9)$$

in the Wolfenstein parametrization and setting the strong phase of C_{15}^T to zero, *i.e.*, the EWP contribution is $\sim O(2\%)$ and can thus be neglected in

the decay amplitude $B^+ \rightarrow \pi^0 \pi^+$.⁵ Thus, C_{15}^T can be determined directly from the decay rate $B^+ \rightarrow \pi^+ \pi^0$.

Similarly, the expression for $A_{3/2}$ (Eqn.(4)) simplifies to

$$A_{3/2} \approx -8 C_{15}^T \left(|\lambda_u^{(s)}| e^{i\gamma} - |\lambda_c^{(s)}| \frac{3}{2} \kappa \right) \quad (10)$$

so that

$$|A_{3/2}| = 8 C_{15}^T |\lambda_u^{(s)}| \sqrt{(1 + \delta_{EW}^2 - 2\delta_{EW} \cos \gamma)}, \quad (11)$$

where δ_{EW} is given by $|\lambda_c^{(s)}|/|\lambda_u^{(s)}| \frac{3}{2} \kappa \sim O(1)$, *i.e.*, in this case, due to the CKM factors, the EWP contribution $\propto \lambda_c^{(s)}$ is important. Thus, knowing C_{15}^T from the decay rate $B^+ \rightarrow \pi^+ \pi^0$ and γ , we can determine $A_{3/2}$ and conversely γ can be determined if the (magnitude) of $A_{3/2}$ is known using some other method and the decay rate $B^+ \rightarrow \pi^0 \pi^+$ is measured. In using this relation, it is crucial that the parameter δ_{EW} is calculable.

In the analysis up to now, annihilation contributions are included.

2 Method 1

The decay amplitudes for B_d , B^+ decays to $\eta_8 = 1/\sqrt{6} (2 s\bar{s} - u\bar{u} - d\bar{d})$ and $\eta_1 = 1/\sqrt{3} (s\bar{s} + u\bar{u} + d\bar{d})$ can be written as [6]

$$\begin{aligned} \sqrt{6}\mathcal{A}(B^+ \rightarrow K^+ \eta_8) &= \lambda_u^{(s)} \tilde{C}_3^T - \lambda_c^{(s)} C_3^P - \lambda_u^{(s)} \tilde{C}_6^T + \lambda_c^{(s)} C_6^P \\ &\quad - 9 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) + 3 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right), \\ \sqrt{3}\mathcal{A}(B^+ \rightarrow K^+ \eta_1) &= 2 \left(\lambda_u^{(s)} \tilde{C}_3^T - \lambda_c^{(s)} C_3^P \right) + \lambda_u^{(s)} \tilde{C}_6^T - \lambda_c^{(s)} C_6^P \\ &\quad + 3 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) + 6 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) \\ &\quad + 3 \left(\lambda_u^{(s)} \tilde{E}_3^T - \lambda_c^{(s)} E_3^P \right) + 3 \left(\lambda_u^{(s)} \tilde{D}_6^T - \lambda_c^{(s)} D_6^P \right) \\ &\quad + 9 \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right), \end{aligned} \quad (12)$$

⁵This EWP contribution can actually be included [5], but we neglect it for simplicity here.

$$\begin{aligned}
\sqrt{6}\mathcal{A}(B_d \rightarrow K^0\eta_8) &= \lambda_u^{(s)}\tilde{C}_3^T - \lambda_c^{(s)}C_3^P + \lambda_u^{(s)}\tilde{C}_6^T - \lambda_c^{(s)}C_6^P \\
&\quad - 5 \left(\lambda_u^{(s)}\tilde{C}_{15}^T - \lambda_c^{(s)}C_{15}^P \right) - \left(\lambda_u^{(s)}\tilde{A}_{15}^T - \lambda_c^{(s)}A_{15}^P \right), \\
\sqrt{3}\mathcal{A}(B_d \rightarrow K^0\eta_1) &= 2 \left(\lambda_u^{(s)}\tilde{C}_3^T - \lambda_c^{(s)}C_3^P \right) - \lambda_u^{(s)}\tilde{C}_6^T + \lambda_c^{(s)}C_6^P \\
&\quad - \left(\lambda_u^{(s)}\tilde{C}_{15}^T - \lambda_c^{(s)}C_{15}^P \right) - 2 \left(\lambda_u^{(s)}\tilde{A}_{15}^T - \lambda_c^{(s)}A_{15}^P \right) \\
&\quad + 3 \left(\lambda_u^{(s)}\tilde{E}_3^T - \lambda_c^{(s)}E_3^P \right) - 3 \left(\lambda_u^{(s)}\tilde{D}_6^T - \lambda_c^{(s)}D_6^P \right) \\
&\quad - 3 \left(\lambda_u^{(s)}\tilde{D}_{15}^T - \lambda_c^{(s)}D_{15}^P \right), \tag{13}
\end{aligned}$$

where E_3 , D_6 and D_{15} are the amplitudes which contribute only to B meson decays to a final state involving η_1 [6].⁶ D_6 and D_{15} are annihilation amplitudes. We assume that the mass eigenstates η and η' are given by the canonical mixing⁷:

$$\begin{aligned}
\eta &= \frac{2\sqrt{2}}{3}\eta_8 - \frac{1}{3}\eta_1 \\
&= \frac{1}{\sqrt{3}}(s\bar{s} - u\bar{u} - d\bar{d}), \\
\eta' &= \frac{1}{3}\eta_8 + \frac{2\sqrt{2}}{3}\eta_1 \\
&= \frac{1}{\sqrt{6}}(2s\bar{s} + u\bar{u} + d\bar{d}). \tag{14}
\end{aligned}$$

This mixing is consistent with the present data [9]. Then, the decay amplitudes for B^+ , B_d decays to η and η' are

$$\begin{aligned}
-\sqrt{3}\mathcal{A}(B^+ \rightarrow K^+\eta) &= \lambda_u^{(s)}\tilde{C}_6^T - \lambda_c^{(s)}C_6^P + 7 \left(\lambda_u^{(s)}\tilde{C}_{15}^T - \lambda_c^{(s)}C_{15}^P \right) \\
&\quad + \left(\lambda_u^{(s)}\tilde{E}_3^T - \lambda_c^{(s)}E_3^P \right) + \left(\lambda_u^{(s)}\tilde{D}_6^T - \lambda_c^{(s)}D_6^P \right) \\
&\quad + 3 \left(\lambda_u^{(s)}\tilde{D}_{15}^T - \lambda_c^{(s)}D_{15}^P \right), \\
\sqrt{6}\mathcal{A}(B^+ \rightarrow K^+\eta') &= 3 \left(\lambda_u^{(s)}\tilde{C}_3^T - \lambda_c^{(s)}C_3^P \right) + \lambda_u^{(s)}\tilde{C}_6^T - \lambda_c^{(s)}C_6^P
\end{aligned}$$

⁶The notation \tilde{D}^T , D_P is similar to \tilde{C}^T , C_P (Eqn.(2)).

⁷Both the methods can be easily modified in the case of a general mixing angle, provided the mixing angle is known.

$$\begin{aligned}
& + \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) + 9 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) \\
& + 4 \left(\lambda_u^{(s)} \tilde{E}_3^T - \lambda_c^{(s)} E_3^P \right) + 4 \left(\lambda_u^{(s)} \tilde{D}_6^T - \lambda_c^{(s)} D_6^P \right) \\
& + 12 \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right), \tag{15}
\end{aligned}$$

$$\begin{aligned}
-\sqrt{3} \mathcal{A}(B_d \rightarrow K^0 \eta) &= -\lambda_u^{(s)} \tilde{C}_6^T + \lambda_c^{(s)} C_6^P + 3 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) \\
&+ \left(\lambda_u^{(s)} \tilde{E}_3^T - \lambda_c^{(s)} E_3^P \right) - \left(\lambda_u^{(s)} \tilde{D}_6^T - \lambda_c^{(s)} D_6^P \right) \\
&- \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right), \\
\sqrt{6} \mathcal{A}(B_d \rightarrow K^0 \eta') &= 3 \left(\lambda_u^{(s)} \tilde{C}_3^T - \lambda_c^{(s)} C_3^P \right) - \lambda_u^{(s)} \tilde{C}_6^T + \lambda_c^{(s)} C_6^P \\
&- 3 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) - 3 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) \\
&+ 4 \left(\lambda_u^{(s)} \tilde{E}_3^T - \lambda_c^{(s)} E_3^P \right) - 4 \left(\lambda_u^{(s)} \tilde{D}_6^T - \lambda_c^{(s)} D_6^P \right) \\
&- 4 \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right). \tag{16}
\end{aligned}$$

From Eqns.(3), (12) and (14), we get the relation [2]

$$\begin{aligned}
\sqrt{6} \mathcal{A}(B^+ \rightarrow K^+ \eta_8) &= \frac{4}{3} \sqrt{3} \mathcal{A}(B^+ \rightarrow K^+ \eta) + \frac{1}{3} \sqrt{6} \mathcal{A}(B^+ \rightarrow K^+ \eta') \\
&= 2 \mathcal{A}(B^+ \rightarrow \pi^+ K^0) + \sqrt{2} \mathcal{A}(B^+ \rightarrow \pi^0 K^+). \tag{17}
\end{aligned}$$

As mentioned earlier, the magnitude of $A_{3/2}$ is known as a function of γ in terms of the $B^+ \rightarrow \pi^0 \pi^+$ decay rate (Eqn.(11)). This, for a given γ , we can construct the two triangles formed by $B_d \rightarrow \pi K$, $A_{3/2}$ and $B^+ \rightarrow \pi K$, $A_{3/2}$ (corresponding to Eqns.(4) and Δ 's DEB and ADB respectively in Fig.1) and the quadrangle formed by $B^+ \rightarrow \pi K$, $K^+ \eta$, $K^+ \eta'$ corresponding to Eqn.(17) ($ADFC$ of Fig.1).⁸ Thus, we know the phases (in the convention where the phase of C_{15}^T is zero) of the decay amplitudes $B^+ \rightarrow \pi K$, ηK^+ ,

⁸There are two discrete ambiguities in the construction of Fig.1: Δ 's ADB and DEB can be on the same side of the common base DB and similarly in the quadrangle $ADFC$ the vertices A and F can be on the same side of the diagonal DC .

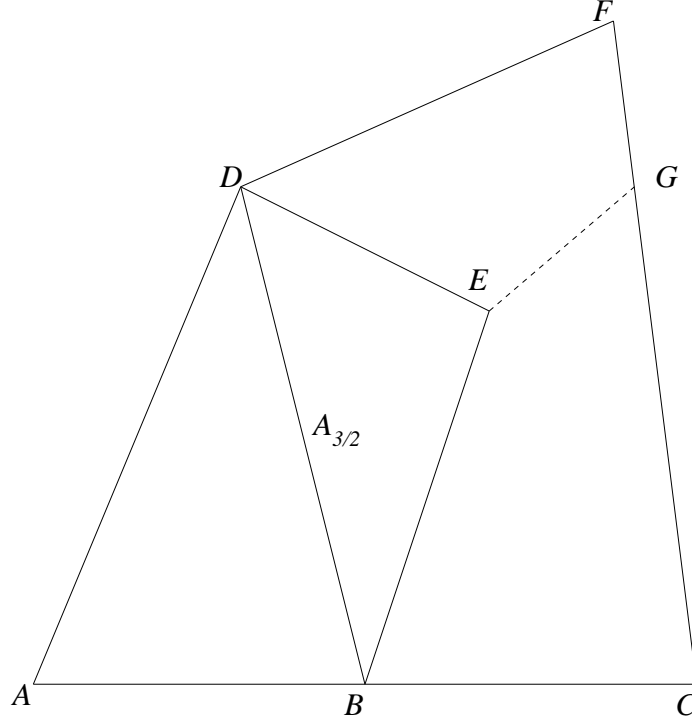


Figure 1: The polygon construction for Method 1. $AB = BC = |\mathcal{A}(B^+ \rightarrow \pi^+ K^0)|$, $AD = \sqrt{2} |\mathcal{A}(B^+ \rightarrow \pi^0 K^+)|$, $DF = \frac{1}{3}\sqrt{6} |\mathcal{A}(B^+ \rightarrow K^+ \eta')|$, $GC = 3 FG = \sqrt{3} |\mathcal{A}(B^+ \rightarrow K^+ \eta)|$, $DE = \sqrt{2} |\mathcal{A}(B_d \rightarrow \pi^0 K^0)|$, $BE = |\mathcal{A}(B_d \rightarrow \pi^- K^+)|$. Given γ and the $B^+ \rightarrow \pi^+ \pi^0$ decay rate we know $DB = |A_{3/2}|$ from Eqn.(11). The prediction for $\sqrt{3} |\mathcal{A}(B_d \rightarrow K^0 \eta)|$ is EG .

$K^+\eta'$ and $B_d \rightarrow \pi K$ as a function of γ from this construction (the magnitudes are, of course, known from the measurement of the decay rates).

We have included the annihilation contributions up to now.

If we *neglect* the annihilation amplitudes, A_{15} , D_6 and D_{15} , using Eqns.(1), (15) and (16), we get the relations:

$$\begin{aligned} -\sqrt{3}\mathcal{A}(B_d \rightarrow K^0\eta) &= -\sqrt{2}\mathcal{A}(B_d \rightarrow \pi^0 K^0) + \frac{1}{3}\sqrt{6}(B^+ \rightarrow K^+\eta') \\ &\quad + \frac{1}{3}\sqrt{3}\mathcal{A}(B^+ \rightarrow K^+\eta), \end{aligned} \quad (18)$$

$$\begin{aligned} \sqrt{6}\mathcal{A}(B_d \rightarrow K^0\eta') &= -\sqrt{2}\mathcal{A}(B_d \rightarrow \pi^0 K^0) + \frac{4}{3}\sqrt{6}(B^+ \rightarrow K^+\eta') \\ &\quad + \frac{4}{3}\sqrt{3}\mathcal{A}(B^+ \rightarrow K^+\eta). \end{aligned} \quad (19)$$

Thus, we can predict the decay amplitudes $B_d \rightarrow K^0\eta$, $K^0\eta'$ as a function of γ since, as mentioned above, we know the magnitudes and phases (the latter from Fig.1) of all the other decay amplitudes in Eqns.(18) and (19). In fact, the decay amplitude $B_d \rightarrow K^0\eta$ is shown as *EG* in Fig.1. Once the decay rate $B_d \rightarrow K^0\eta$ or $K^0\eta'$ is measured, γ can be determined. Thus, γ can be determined (up to a four-fold discrete ambiguity) by the measurement of the decay rates for 8 modes – $B^+ \rightarrow \pi^0\pi^+$, πK , $K^+\eta$, $K^+\eta'$, $B_d \rightarrow \pi K$, $K^0\eta$ (or $K^0\eta'$).

3 Method 2

This method is based on the method of Gronau *et al.* [1]. In [1], the annihilation amplitudes are neglected *and* the relation $C_3^T - C_6^T - C_{15}^T = 0$ is assumed. In other words, rescattering effects are neglected so that the amplitude for the decay $B^+ \rightarrow K^0\pi^+$ has no weak phase $e^{i\gamma}$. We neglect the annihilation amplitudes, but do not assume any relation between the C 's. Thus, we include partly the rescattering effects. The decay amplitudes for

$B_s \rightarrow \pi^0 \eta_8, \eta_1$ are [6]

$$\begin{aligned} \sqrt{3}\mathcal{A}(B_s \rightarrow \pi^0 \eta_8) &= 2 \left(\lambda_u^{(s)} \tilde{C}_6^T - \lambda_c^{(s)} C_6^P \right) - 4 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) \\ &\quad + 4 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right), \end{aligned} \quad (20)$$

$$\begin{aligned} \sqrt{6}\mathcal{A}(B_s \rightarrow \pi^0 \eta_1) &= 2 \left(\lambda_u^{(s)} \tilde{C}_6^T - \lambda_c^{(s)} C_6^P \right) - 4 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) \\ &\quad - 8 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) + 6 \lambda_u^{(s)} \tilde{D}_6^T - 6 \lambda_c^{(s)} D_6^P \\ &\quad - 12 \lambda_u^{(s)} \tilde{D}_{15}^T + 12 \lambda_c^{(s)} D_{15}^P. \end{aligned} \quad (21)$$

With the canonical mixing (Eqn.14)), we get

$$\begin{aligned} \sqrt{6}\mathcal{A}(B_s \rightarrow \pi^0 \eta) &= 2 \left(\lambda_u^{(s)} \tilde{C}_6^T - \lambda_c^{(s)} C_6^P \right) - 4 \left(\lambda_u^{(s)} \tilde{C}_{15}^T - \lambda_c^{(s)} C_{15}^P \right) \\ &\quad + 8 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) - 2 \left(\lambda_u^{(s)} \tilde{D}_6^T - \lambda_c^{(s)} D_6^P \right) \\ &\quad + 4 \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right), \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{A}(B_s \rightarrow \pi^0 \eta') &= \sqrt{2}(B_s \rightarrow \pi^0 \eta) - 2\sqrt{3} \left[2 \left(\lambda_u^{(s)} \tilde{A}_{15}^T - \lambda_c^{(s)} A_{15}^P \right) \right. \\ &\quad \left. - \lambda_u^{(s)} \tilde{D}_6^T + \lambda_c^{(s)} D_6^P + 2 \left(\lambda_u^{(s)} \tilde{D}_{15}^T - \lambda_c^{(s)} D_{15}^P \right) \right]. \end{aligned} \quad (23)$$

Neglecting the annihilation amplitudes, A_{15} , D_6 and D_{15} , from Eqns.(1), (3), (22) and (23) we get the relations [1]:

$$\begin{aligned} \sqrt{6}\mathcal{A}(B_s \rightarrow \pi^0 \eta) &\approx -\mathcal{A}(B^+ \rightarrow K^0 \pi^+) + \sqrt{2}\mathcal{A}(B_d \rightarrow K^0 \pi^0) \\ &= \sqrt{2}\mathcal{A}(B^+ \rightarrow K^+ \pi^0) - \mathcal{A}(B_d \rightarrow K^+ \pi^-), \end{aligned} \quad (24)$$

$$\sqrt{6}\mathcal{A}(B_s \rightarrow \pi^0 \eta) \approx \sqrt{3}\mathcal{A}(B_s \rightarrow \pi^0 \eta'). \quad (25)$$

From Eqns.(24) and (25) we see that the decay amplitudes $B_d \rightarrow \pi K$, $B^+ \rightarrow \pi K$ form the sides and $B_s \rightarrow \pi \eta$ (or η') the diagonal of a quadrangle shown in Fig.2. Thus, the measurement of these 5 decay rates fixes this quadrangle ⁹ of which the other diagonal is $A_{3/2}$ (see Eqns.(4) and Fig.2) [1].

⁹ There is a discrete ambiguity in this construction since Δ 's ABC and ADC can be on the same side of the common base AC in Fig.2.

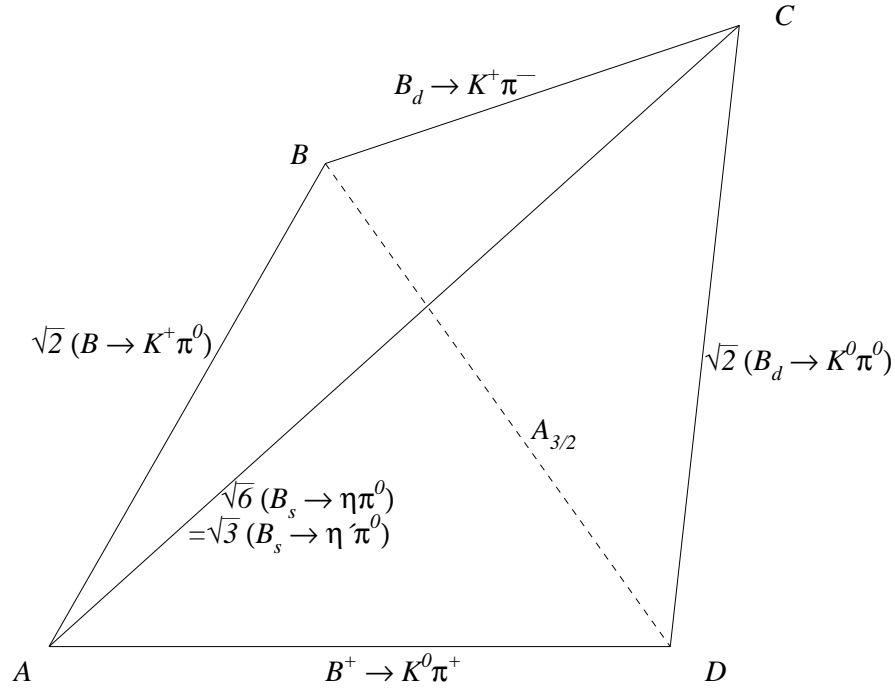


Figure 2: The polygon construction for Method 2. Knowing $|A_{3/2}|$ from this Figure, we can determine γ using Eqn.(11) if the decay rate $B^+ \rightarrow \pi^0 \pi^+$ is also measured.

Knowing the magnitude of $A_{3/2}$ from Fig.2 and the decay rate $B^+ \rightarrow \pi^0 \pi^+$, we can determine γ from Eqn.(11). Thus, γ can be determined (up to a two-fold discrete ambiguity) by the measurement of the decay rates for 6 modes – $B^+ \rightarrow \pi^0 \pi^+$, πK , $B_d \rightarrow \pi K$, and $B_s \rightarrow \pi^0 \eta$ (or $\pi^0 \eta'$). In the method of [1], measurement of the rates for the CP-conjugate processes of all the above modes is also required to determine γ .

4 Discussions

We comment on the accessibility of the various decay modes used in the two methods. The B_d and B^+ decay modes should be accessible at the e^+e^- machines whereas the $B_s \rightarrow \pi \eta$ (η') decay mode will only be accessible at a hadron machine. Since the QCD penguin does not contribute to this B_s decay, the decay rate is expected to be small. The measurements of the decay rates to CP eigenstate final states: $B_s \rightarrow \pi \eta$ (η'), $B_d \rightarrow \pi^0 K^0$ and $B_d \rightarrow K^0 \eta$ (η') require external tagging.

As mentioned earlier, we have used flavor $SU(3)$ symmetry in both the methods. In the factorization approximation, $SU(3)$ breaking in the tree level amplitudes can be incorporated by factors of f_K/f_π (see, for example, [1]). For example, $C_{15}^T (\Delta S = 1) = f_K/f_\pi \times C_{15}^T (\Delta S = 0)$. However, since some of the strong penguin operators are $(V - A) \times (V + A)$, in the penguin amplitudes, the $SU(3)$ breaking effects are difficult to estimate, but the breaking will still be less than $\sim O(30\%)$. In method 2, we use the decay mode $B_s \rightarrow \pi \eta$ (η') which does not have the QCD penguin contribution, but does have the EWP contribution. The EWP operators $\mathcal{O}_{7,8}$ have very small WC's whereas the EWP operators with significant WC's, $\mathcal{O}_{9,10}$, are Fierz-equivalent to the tree level operators $\mathcal{O}_{1,2}$ [8]. So in the factorization approximation, the corrections due to $SU(3)$ breaking in relating the penguin amplitudes for $B_s \rightarrow \pi \eta$ (η') to the ones for B_d , $B^+ \rightarrow \pi K$ are given by factors of $f_{\eta, \eta'}/f_K$.

We have also assumed that the $SU(3)$ breaking in the strong phases is small. A possible justification is that at the energies of the final state particles $\sim m_b/2$, the phase shifts are not expected to be sensitive to the $SU(3)$ breaking given by, say, $m_K - m_\pi$ (which is much smaller than the final state momenta). However, it is hard to quantify this effect.

Both the methods can be used with CP-conjugates of all the decay modes as well. We have neglected annihilation amplitudes: A_{15} , D_6 and D_{15} . The validity of this assumption can be checked by comparing the decay rates $B_s \rightarrow \pi\eta$ and $B_s \rightarrow \pi\eta'$ – these two decay amplitudes differ only in the annihilation contribution (see Eqn.(23)). In the absence of significant annihilation contribution, the decay rate for $B_s \rightarrow \pi\eta'$ should be twice that for $B_s \rightarrow \pi\eta$.

In summary, we have discussed two new methods (based on flavor $SU(3)$ symmetry) to determine the weak phase γ using the decays B_d , $B^+ \rightarrow K\eta$ (η') and $B_s \rightarrow \pi\eta$ (η'), respectively. These methods partly take into account rescattering effects.

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